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Contributions of strange quarks to the magnetic moment of the proton

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Abstract

Using the Gerasimov-Drell-Hearn sum rule and experimental total cross sections for photoproduction of η , K and ϕ mesons on the proton, we obtain upper bounds for the contribution of strange quarks to the anomalous magnetic moment of the proton, $|\kappa_s| \leq 0.20 \kappa_p$.

The proposed experiments to measure the spin-dependence of the absorption cross section are expected to lower these bounds considerably. The existing data on η production and phenomenological models for K production agree with a negative sign for κ_s .

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The strange vector form factors of the nucleon are an interesting and challenging topic in medium energy physics. They obtain contributions only from non-valence quarks and allow for a direct study of the role played by strange quarks for the nucleon's response to low- and intermediate-energy probes [1]. Considerable experimental activities have been devoted to measure these form factors by means of parity violating electron scattering at MAMI, MIT/Bates, and Jefferson Lab (formerly CEBAF) [2]. At the same time, a wealth of model calculations is now available [3]. However, these calculations involve large theoretical uncertainties, because sea quark effects arise from a subtle interplay of quantum fluctuations which are difficult to describe. Recently, there has been some effort to reduce the uncertainty in these predictions by using chiral perturbation theory [4] and dispersion theory in conjunction with unitarity [5]. However, the aim of this letter is to point out a complementary way to estimate the importance of the strange quark for the structure of the nucleon.

As is well known, the ground state properties of the nucleon are connected with its excitation spectrum via dispersion relations and low energy theorems. In particular, the Gerasimov-Drell-Hearn (GDH) sum rule [6] relates the anomalous magnetic moment κ of the nucleon to the difference of its polarized total photoabsorption cross sections,

$$-\frac{\kappa^2}{4} = \frac{m^2}{8\pi^2\alpha} \int_0^\infty \frac{d\nu}{\nu} (\sigma_{1/2}(\nu) - \sigma_{3/2}(\nu)) =: I(Q^2 = 0) , \quad (1)$$

where $\sigma_{3/2}$ and $\sigma_{1/2}$ denote the cross sections for the two possible combinations of the spins of nucleon and photon, α is the fine structure constant, ν the photon energy in the laboratory frame and m the mass of the nucleon. This sum rule is based on Lorentz and gauge invariance, crossing symmetry, causality and unitarity. The only further assumption in deriving eq. (1) is the convergence of the integral. There are in fact very strong arguments for such a convergence from the Froissart bound for spin dependent processes in QCD [7]. A general review of the field can be found, e.g. in [8]. In particular, we note that the sum rule can be extended to virtual photons. The difference of the helicity cross sections is then related to the transverse-transverse structure function

$$\sigma_{TT'} = \frac{\sigma_{3/2} - \sigma_{1/2}}{2} , \quad (2)$$

which can be measured with longitudinally polarized electrons and nucleons polarized in the direction of the exchanged virtual photon,

$$I(Q^2) = -\frac{m^2}{4\pi^2\alpha} \int_0^\infty \frac{d\nu}{\nu} (1-x) \sigma_{TT'}(\nu, Q^2) , \quad (3)$$

where $x = \frac{Q^2}{2m\nu}$ denotes the Bjorken scaling variable and $-Q^2$ is the 4-momentum transfer of the virtual photon. $\sigma_{TT'}$ may be expressed through a multipole decomposition. Up to kinematical factors, one has

$$\sigma_{TT'} \propto \left(-|E_{0+}|^2 - 3|E_{1+}|^2 + |M_{1+}|^2 - |M_{1-}|^2 - 6E_{1+}^*M_{1+} + \dots \right) , \quad (4)$$

where the dots correspond to higher multipoles [8].

Our aim is to use the GDH sum rule and experimental photoproduction data from a proton target to estimate the fraction of the magnetic moment of the proton which is due to the strangeness degree of freedom. Therefore, we will study the contributions of photoproduction of the lightest mesons carrying strange quarks (η, K, ϕ) to the integral on the right hand side of eq.(1). Obviously the contributions of the various reaction channels add incoherently to the partial cross sections in eq. (1), and the sum over all channels involving reaction products with strange quarks gives some measure of the influence of strangeness on the magnetic moment of the nucleon.

In phenomenological analyses [9], it has been found that the GDH integral for the proton is already saturated for photon energies at $\nu_{max} \approx 2$ GeV,

$$\kappa_p^2 \approx \frac{m^2}{2\pi^2\alpha} \int_0^{\nu_{max}} \frac{d\nu}{\nu} (\sigma_{3/2}(\nu) - \sigma_{1/2}(\nu)) . \quad (5)$$

Since the existing data base is not yet accurate enough to extract $\sigma_{3/2}$ and $\sigma_{1/2}$ for channels involving strange quarks, we use the total cross section as an upper bound. In the electroproduction formalism this corresponds to using the structure function

$$\sigma_T = \frac{\sigma_{3/2} + \sigma_{1/2}}{2} \quad (6)$$

instead of $\sigma_{TT'}$. With $\sigma_{3/2}$ and $\sigma_{1/2}$ being positive definite quantities and $\sigma_{tot}(\nu) = 2\sigma_T(\nu, Q^2 = 0)$, we have

$$-\sigma_{tot}(\nu) \leq \sigma_{3/2}(\nu) - \sigma_{1/2}(\nu) \leq \sigma_{tot}(\nu) , \quad (7)$$

and

$$\kappa_p^2 \lesssim \frac{m^2}{2\pi^2\alpha} \int_0^{\nu_{max}} \frac{d\nu}{\nu} \sigma_{tot}(\nu) . \quad (8)$$

We will use eq. (8) to estimate the contribution of reactions involving strange quarks to the anomalous magnetic moment of the proton, and denote this contribution by κ_s . The experimental data for the total cross sections for photoproduction of η, K and ϕ mesons on the nucleon are extracted from refs. [10, 11], [12] and [10, 13], respectively. A summary of the older data can be found in ref. [14]. In the low energy region the experimental data have been interpolated, whereas at high energies a constant cross section equal to the value at the highest measured energy has been assumed. In the case of K production we have to sum over the reactions $\gamma p \rightarrow K^+\Sigma^0, K^+\Lambda$ and $K^0\Sigma^+$. For the ϕ and η mesons which carry no net strangeness, we explicitly consider their flavour content. The ϕ is an almost pure $|\bar{s}s\rangle$ state, since $\phi - \omega$ mixing is small ($\epsilon \approx 0.055$). However, in the eta case the octet and singlet states mix with an angle $\theta_{\eta\eta'} \approx -0.35$ leading to a flavour content $|\eta\rangle = 3^{-1/2}(|\bar{u}u\rangle + |\bar{d}d\rangle - |\bar{s}s\rangle)$. Since we are only interested in the contribution

ν_{max} [GeV]	$\kappa_s^2(\phi)$	$\kappa_s^2(\eta)$	$\kappa_s^2(K)$	κ_s^2 (sum)	κ_s^2/κ_p^2 [%]	$ \kappa_s /\kappa_p$ [%]
2.0	0.00	0.03	0.04	0.07	2	14
5.0	0.01	0.03	0.06	0.10	3	17
50.0	0.03	0.03	0.10	0.16	5	22

Table 1: The contributions of ϕ , η , and K production to the anomalous magnetic moment of the proton ($\kappa_p = 1.79$), for different cut-offs ν_{max} .

of strange quarks to the magnetic moment of the proton, we multiply the η contribution by a factor 1/3. In Table 1 we display the results obtained with the three different cutoff values $\nu_{max} = 2.0, 5.0$ and 50.0 GeV. Since the thresholds for η, K and ϕ production are considerably higher than the pion threshold, it is reasonable to integrate at least up to $\nu_{max} = 5.0$ GeV. In particular, the contribution of ϕ production is completely negligible for $\nu_{max} = 2.0$ GeV. Of course, the largest value in the table, $\nu_{max} = 50.0$ GeV, is not very realistic since at such energies single meson production is negligible and the approximation of a constant cross section certainly overestimates the effect. However, the last column in Table 1 shows that the final results are not very sensitive to the cut-off energy. The dependence on ν_{max} is displayed in Fig. 1 for both the individual contributions and the sum. The logarithmic increase at high energies is due to the assumption of a constant cross section in this region. It can be noticed that even very unrealistic values of ν_{max} do not change the order of magnitude of the estimate. From Table 1 we conclude that $\kappa_s^2 \leq 0.04 \kappa_p^2$ is a reasonable upper bound, i.e. $|\kappa_s| \leq 0.20 \kappa_p$. This bound is in agreement with most of the theoretical calculations [3, 4, 5]. In our calculation, K production is the most important reaction, whereas the contributions of ϕ and η mesons are considerably smaller.

Although our approach relies mainly on experimental data, it has some flaws, and the upper bound on $|\kappa_s|$ should be considered as order of magnitude estimate only. In particular, the production of two- and more-meson states should lead to an enhancement at the higher energies. On the other hand, the RHS of eq. (8) is expected to overestimate the GDH integral by far, because the GDH integrand is not positive definite and cancellations can occur. This effect can be studied by evaluating the GDH integral within hadronic models. In the case of η photoproduction, phenomenological models [15] predict a negative contribution to κ_p^2 of about the same magnitude as our bound. This result can be understood by means of eq. (4). Photoproduction of η mesons is dominated by the S_{11} resonance which has a 50% branching ratio to ηN , and the corresponding E_{0+} multipole enters with a negative sign in eq. (4). For the numerically more important contribution of K mesons, we evaluate the GDH integral using the model of Mart et al. [16]. In Fig. 2, we show a comparison of the transverse structure function σ_T (eq. (6))

and the transverse-transverse structure function $\sigma_{TT'}$ (eq. (2)) for the $K^+\Lambda$ - and $K^+\Sigma^0$ -channels. It is clearly seen that σ_T overestimates $\sigma_{TT'}$ considerably. Moreover, the model calculation predicts a negative sign for $\sigma_{TT'}$ (Note that $-\sigma_{TT'}$ has been plotted in Fig. 2). This sign follows from the fact that the model predicts a dominance of the multipoles E_{0+} and M_{1-} (see eq. (4)). Since there are only few data on kaon photoproduction off the neutron, we are not able to perform the isospin decomposition of eq. (5) on the basis of experiments,

$$\kappa_{p/n}^2 = (\kappa_{I=1} \pm \kappa_{I=0})^2 = \kappa_{I=1}^2 \pm 2 \kappa_{I=1} \kappa_{I=0} + \kappa_{I=0}^2 . \quad (9)$$

Obviously, the strange quark contributes only to the isoscalar magnetic moment. Since the GDH integral for $\kappa_{I=1}^2$ is well saturated by pion photoproduction [9], we do not expect substantial contributions to $\kappa_{I=1}^2$ from reactions involving ϕ , η and K mesons. On the other hand, the remaining two terms are not well described by the pion data and additional contributions are needed. Present estimates for the isoscalar-isovector interference cannot even describe the sign of this term properly. With our model [16], we are in the position to isolate the mixed term by subtracting κ_p^2 and κ_n^2 . Integrating eq. (1) up to $\nu_{max} = 2.2$ GeV, we find $\kappa_p^2(K) = -0.07$ in qualitative agreement with Table 1. However, the value $\kappa_n^2(K) = -0.02$ indicates that kaon photoproduction contains contributions to both $\kappa_{I=1}^2 + \kappa_{I=0}^2$ and $2 \kappa_{I=1} \kappa_{I=0}$, namely -0.045 and -0.025 , respectively. We conclude, however, that the contribution of the K agrees with a negative sign for $\kappa_{I=0}$ and κ_s .

In the future, it may be expected that the results of the Bonn-Mainz GDH collaboration and similar experiments at Jefferson Lab will lead to much more stringent bounds [17, 18]. In this context it will also be interesting to study the dependence of the GDH and the Burkhardt-Cottingham (BC) sum rules on the momentum transfer. The BC sum rule [19] relates the integral over the longitudinal-transverse structure function $\sigma_{LT'}$ with ground state properties of the nucleon [8]. Both sum rules have never been measured. They involve experiments with longitudinally polarized electrons and nucleon polarization in the scattering plane, parallel or perpendicular to the momentum of the virtual photon, respectively. An experiment at Jefferson Lab which has been proposed to measure $\sigma_{TT'}$ and $\sigma_{LT'}$ [18], will determine the structure functions $G_1(\nu, Q^2)$ and $G_2(\nu, Q^2)$ from polarized deep-inelastic electron nucleon scattering at a momentum transfer $Q^2 \approx 0 \dots 2$ GeV 2 . This will make it possible to evaluate the BC sum rule [20],

$$\begin{aligned} J_2(Q^2) &= m^2 \int_0^\infty d\nu G_2(\nu, Q^2) \\ &= \frac{\mu}{4} G_M(Q^2) (\mu G_M(Q^2) - G_E(Q^2)) , \end{aligned} \quad (10)$$

where μ is the total magnetic moment of the nucleon and G_M (G_E) is its magnetic (electric) form factor. Since eq. (10) connects the form factors of the nucleon with the

sum rule as function of Q^2 , it has the potential to give estimates not only on the magnetic moments but also on the radii. For the leading Q^2 behaviour, we obtain

$$J_2(Q^2) = \frac{\mu\kappa}{4} - Q^2 \frac{\mu}{24} \left((\mu + \kappa) \langle r_M^2 \rangle - \langle r_E^2 \rangle \right) + O(Q^4) . \quad (11)$$

As in the case of the GDH sum rule, the different reactions add incoherently, thus defining the contributions of the individual production channels to the anomalous magnetic moment and a particular combination of electric and magnetic charge radii.

We conclude that detailed investigations of the strangeness contributions to the GDH and BC sum rules could give interesting information about strange quark effects in the nucleon. The results would be complementary to the considerable experimental activities at MAMI, MIT/Bates and Jefferson Lab to measure the contribution of strangeness by parity violating electron scattering.

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Figures

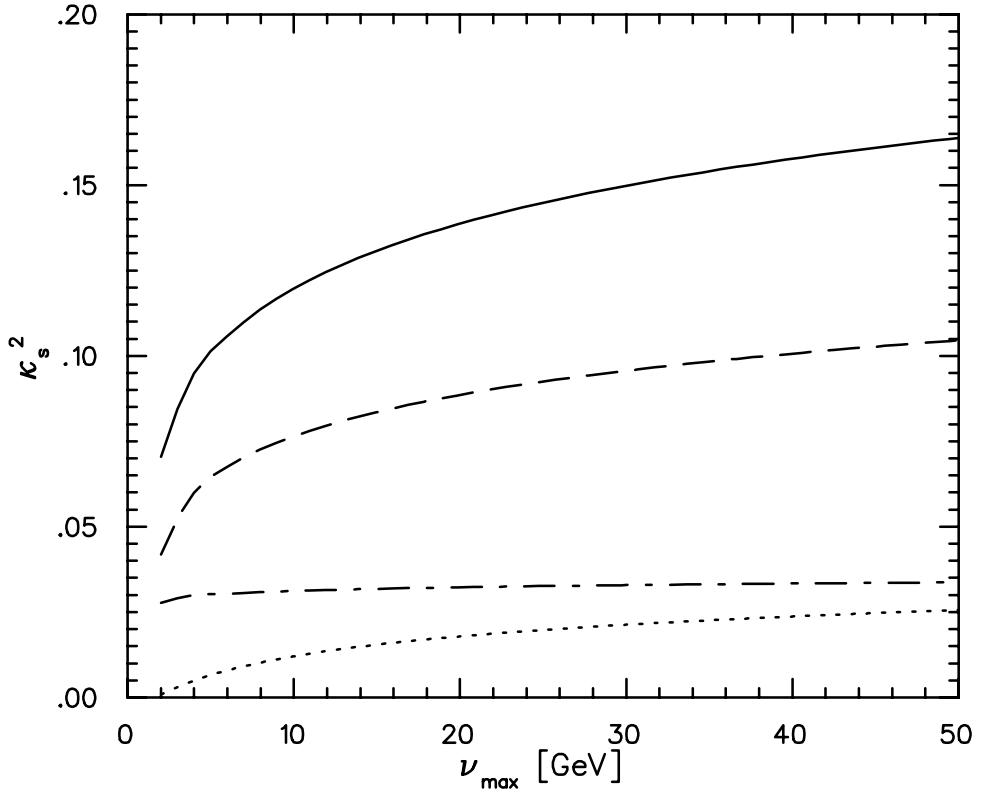


Figure 1: The dependence on ν_{\max} of the contribution of ϕ , η , and K meson photoproduction to the anomalous magnetic moment of the proton (upper bounds). The individual contributions $\kappa_s^2(\phi)$, $\kappa_s^2(\eta)$, $\kappa_s^2(K)$, and the sum κ_s^2 are denoted by the dotted, dash-dotted, dashed, and solid lines, respectively.

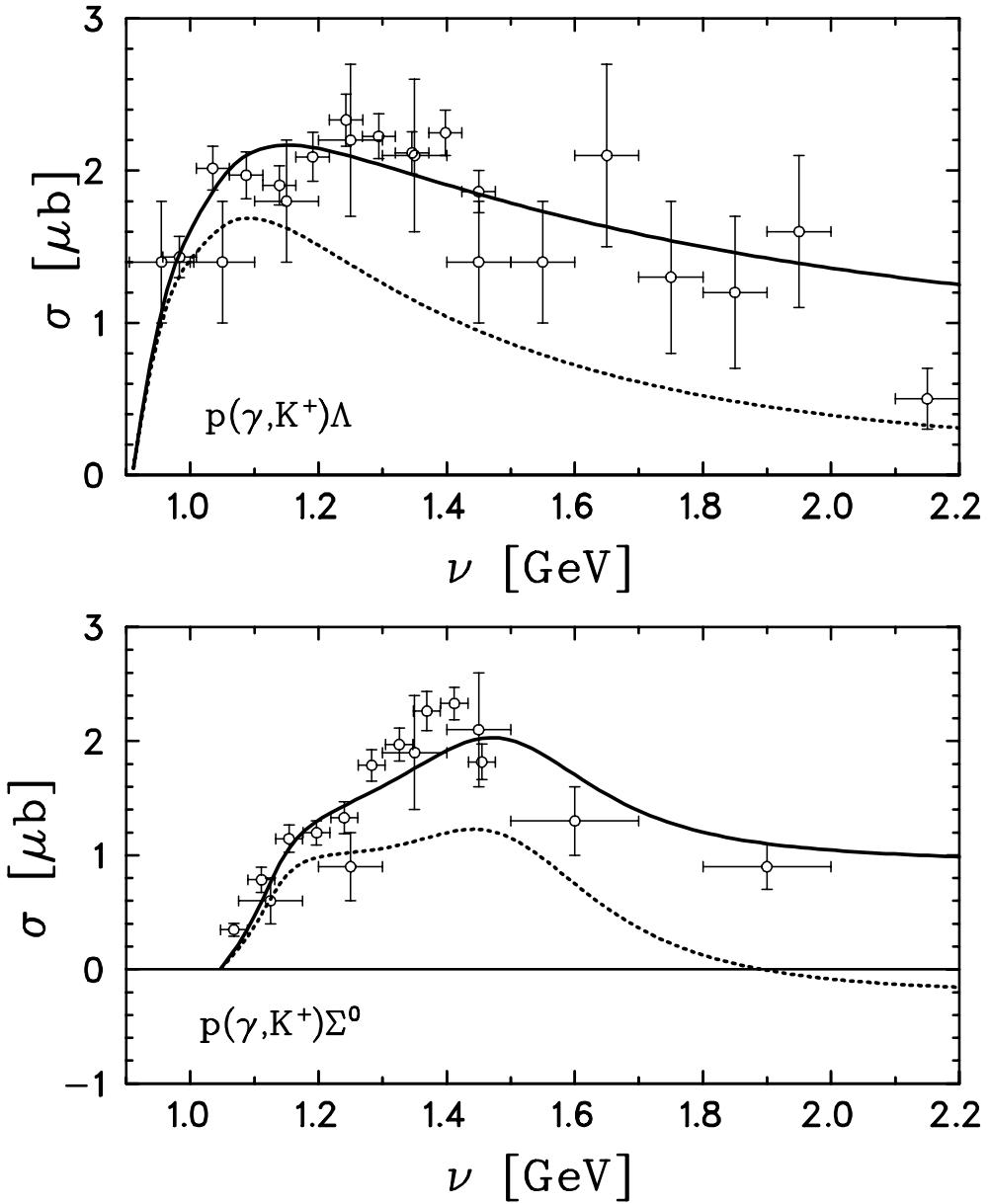


Figure 2: The response functions σ_T (solid lines) and $-\sigma_{TT'}$ (dotted lines) at $Q^2=0$ for the reactions $\gamma+p \rightarrow K^++\Lambda$ (upper panel) and $\gamma+p \rightarrow K^++\Sigma^0$ (lower panel) calculated with the model of Mart et al. [16]. The experimental data points are from Refs. [12].